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Black Hole Entropy in the presence of Chern-Simons Terms

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abstract

We derive a formula for the black hole entropy in theories with gravitational Chern-Simons terms, by generalizing Wald's argument which uses the Noether charge. It correctly reproduces the entropy of three-dimensional black holes in the presence of Chern-Simons term, which was previously obtained via indirect methods.

1 Introduction

In the pursuit of the quantum theory of gravitation, an important guidance comes from the laws of black hole thermodynamics, which can be established by semi-classical arguments. Firstly, manipulation of the classical equation of motion of general relativity yields the first law of black hole dynamics [1] which states

$$\frac{\kappa}{2\pi}\frac{\delta A}{4G_N} = \delta m - \Omega\delta J \tag{1}$$

where A is the horizon area, κ the surface gravity at the horizon, m the mass, Ω the angular velocity and J the angular momentum of the black hole. G_N is the Newton constant entering the Lagrangian as

$$\int d^D x\sqrt{-g}\mathcal{L} = \frac{1}{16\pi G_N}\int d^D x\sqrt{-g}R \tag{2}$$

where D is the number of spacetime dimensions. The law shows a strong resemblance to the first law of thermodynamics if we identify κ with temperature and A with entropy. A key fact is that semi-classical calculation [2] tells us that in the black hole background particles are produced in thermal ensemble at temperature

$$T_H = \frac{\kappa}{2\pi}. \tag{3}$$

Thus it is very natural to set the entropy of a black hole to be

$$S = \frac{A}{4G_N} \tag{4}$$

and to search for its statistical explanation.

A remarkable success is that in superstring theory a class of extremal charged black holes has a dual microscopic description using the D-branes and that it reproduced the entropy (4) in the large charge limit. It also predicted subleading corrections which lead to a deviation from the formula (4), whose existence was also expected from the macroscopic point of view because the low energy limit of string theory includes higher derivative corrections to the Einstein-Hilbert Lagrangian (2). Thankfully Wald and others [3, 4, 5] had already computed the general formula of the black hole entropy of arbitrary general-covariant Lagrangian constructed from the metric, which is given by

$$S = -2\pi \int_\Sigma d^{D-2}x\sqrt{-g}\frac{\delta\mathcal{L}}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}. \tag{5}$$

Here Σ is the horizon cross section and $\epsilon_{\alpha\beta}$ is the binormal to the horizon[1]. Another success of recent advances in quantum gravity is that the deviation calculated from the application of Wald's formula (5) to the superstring effective action agreed precisely with the microscopic entropy calculated using the description as branes. Interested readers can consult the review [6] for details.

A subtlety is that the compactification of superstring theory down to odd dimensional spacetime often includes gravitational Chern-Simons term, and in the case of three dimensions it becomes the topologically massive gravity first described by Deser, Jackiw and Templeton in [7, 8]. It has solutions describing rotating black holes, first discovered by Bañados, Teitelboim and Zanelli [9], but the Wald's formula (5) cannot be applied because it is not manifestly invariant under diffeomorphism. Thus various indirect methods were devised to treat such cases, e.g. [10, 11, 12, 13, 14, 15]. The aim of this short note is to extend the argument in [3] to provide the entropy formula for black holes in the presence of such Chern-Simons terms. We will see that the formula to be obtained reproduces the correction to the entropy of three-dimensional black hole, which was determined before. Our formula is sufficiently general so that it can be applied for mixed gravitational Chern-Simons terms and Green-Schwarz type couplings in any dimensions.

[1] It is defined as $\epsilon_{\alpha\beta} = n_\alpha\xi_\beta - \xi_\alpha n_\beta$ where ξ_α is the null Killing vector generating the horizon and n_α is the ingoing null normal to the horizon cross section normalized to have $n_\alpha\xi^\alpha = -1$.

The rest of the paper is organized as follows. In the next section, we will review the argument in [3, 4, 5], making the necessary changes along the way to accommodate the gravitational Chern-Simons terms. At the end we will obtain the generalized formula. In section 3 we will explicitly evaluate the formula for several examples, and check that it agrees with the result in the literature. We conclude with a short outlook in section 4.

2 General form of the entropy formula

We use a D-form $L(\phi)$ to represent the Lagrangian density, where ϕ collectively denotes fundamental field variables. A crucial property used in the derivation of the first law in [3] is that it satisfies the condition

$$\delta_\xi L(\phi) = \pounds_\xi L(\phi), \tag{6}$$

where δ_ξ is the variation induced by the diffeomorphism by the vector field ξ, while $\pounds_\xi$ denotes the Lie derivative with respect to ξ. The condition (6) is not satisfied for the gravitational Chern-Simons term L_{CS}

$$L_{CS} \sim \mathrm{tr}\,\Gamma \wedge R^{2m-1} + \cdots \tag{7}$$

where Γ is the affine connection, R the curvature two-form and $D = 4m - 1$. It satisfies, however, the condition

$$\delta_\xi L(\phi) = \pounds_\xi L(\phi) + d\,\Xi_\xi. \tag{8}$$

for a suitable $(D-1)$-form Ξ_ξ, which is enough in obtaining generally covariant equations of motion. Here and in the following we define the Lie derivative $\pounds_\xi$ of various non-tensor quantity e.g. Γ to be the resulting expression if the quantity involved were tensorial with the same index structure. We will see below how the first law can be derived starting from the generalized transformation law (8), without assuming the covariance in the strict sense, (6).

First we express the first-order variation of L as

$$\delta L = \sum_\phi E_\phi \delta\phi + d\Theta(\phi, \delta\phi), \tag{9}$$

E_ϕ gives the equation of motion associated with ϕ, while Θ is called the symplectic potential. Let us set

$$\Omega(\phi, \delta_1\phi, \delta_2\phi) = \delta_1\Theta(\phi, \delta_2\phi) - \delta_2\Theta(\phi, \delta_1\phi). \tag{10}$$

It is important that the integral of Ω over a Cauchy surface $\mathcal{C}$, properly projected down to the gauge-invariant phase space, defines the symplectic form in the covariant phase space approach [16, 17].

Secondly, the condition (8) means that the diffeomorphism generated by ξ is a symmetry in the sense of [17], thus the corresponding current j_ξ given by

$$j_\xi = \Theta(\phi, \delta_\xi \phi) - \iota_\xi L - \Xi_\xi \tag{11}$$

is conserved on-shell, i.e. $dj_\xi \simeq 0$. Here $\simeq$ denotes the equality which holds only if the equation of motion is satisfied, and ι_ξ is the interior product. Thus, from the analysis in [18], j_ξ is exact on-shell, i.e. there is a $(D-2)$-form Q_ξ constructed from the products of repeated derivatives of ξ and ϕ such that

$$j_\xi \simeq dQ_\xi. \tag{12}$$

As described in [18], Q_ξ can be constructed algorithmically given the form of j_ξ independently of the detailed structure of the equations of motion.

Before going to the third step, let us define Π_ξ by the equation

$$\delta_\xi \Theta = \pounds_\xi \Theta + \Pi_\xi. \tag{13}$$

Calculating $\delta \delta_\xi L$ in two ways, we obtain

$$d\Pi_\xi \simeq \delta d\, \Xi_\xi. \tag{14}$$

Thus, using the theorem in [18] again, there exists a $(D-2)$-form Σ_ξ such that

$$\Pi_\xi - \delta\, \Xi_\xi \simeq d\Sigma_\xi. \tag{15}$$

Then the third step is the following manipulation of the equation:

$$\begin{aligned} \delta j_\xi &= \delta\Theta(\phi, \delta_\xi \phi) - \iota_\xi \delta L - \delta\, \Xi_\xi \\ &\simeq \delta\Theta(\phi, \delta_\xi \phi) - \pounds_\xi \Theta(\phi, \delta\phi) + d\iota_\xi \Theta - \delta\, \Xi_\xi \\ &= \delta\Theta(\phi, \delta_\xi \phi) - \delta_\xi \Theta(\phi, \delta\phi) + d\iota_\xi \Theta + \Pi_\xi - \delta\, \Xi_\xi \\ &\simeq \Omega(\phi, \delta\phi, \delta_\xi \phi) + d(\iota_\xi \Theta + \Sigma_\xi). \end{aligned} \tag{16}$$

We need to find a quantity C_ξ which satisfies

$$\delta C_\xi = \iota_\xi \Theta + \Sigma_\xi, \tag{17}$$

and let us define

$$Q'_\xi = Q_\xi - C_\xi. \tag{18}$$

Then we finally arrive at the following relation

$$\delta dQ'_\xi \simeq \Omega(\phi, \delta\phi, \delta_\xi \phi), \tag{19}$$

which means that dQ'_ξ is the Hamiltonian generating the diffeomorphism ξ in the covariant phase space approach.

Suppose we have a stationary black hole spacetime with a bifurcate Killing horizon[2] generated by ξ such that $\delta_\xi \phi = 0$. The right hand side of (19) then vanishes. Suppose $\xi = t + \Omega\phi$ where t is the generator of the global time translation, Ω the angular velocity of the horizon and ϕ the angular rotation. Integrating (19) on a Cauchy surface, one obtains

$$\delta \int_\Sigma Q'_\xi \simeq \delta \int_\infty Q'_t + \Omega\, \delta \int_\infty Q'_\phi. \tag{20}$$

where Σ is the horizon cross section and ∞ the asymptotic infinity of the Cauchy surface. Since dQ'_η is the Hamiltonian for the diffeomorphism η, $E = \int_\infty Q'_t$ is the energy which includes the ADM mass and contribution from long range gauge forces, and $J = -\int_\infty Q'_\phi$ is the angular momentum.

In evaluating the left hand side of (20), repeated derivative of ξ can be traded with Riemann curvature times lower derivative of ξ using the Killing identity, thus dQ' can be made to be a linear combination of ξ and $\nabla_a \xi_b$. It is known [4] that we can assume that Σ is the bifurcation surface without changing the integral, and thus $\xi = 0$ and $\nabla_a \xi_b = \kappa \epsilon_{ab}$ on Σ. Let us define, then,

$$S = 2\pi \int_\Sigma Q'_\xi \big|_{\xi\to 0,\ \nabla_a\xi_b \to \epsilon_{ab}}. \tag{21}$$

Now the equation (20) becomes the first law which states:

$$\kappa \delta S \simeq \delta E - \Omega \delta J. \tag{22}$$

The algorithm which determines Q' from L is linear. Thus in finding the contribution to the entropy from the Chern-Simons term, we can follow the procedure described above for L being solely given by the Chern-Simons term, and add the result to the contribution from covariant terms. Unfortunately we have not been able to obtain a general formula similar to the celebrated

[2] A bifurcate horizon is a pair of Killing horizons generated by the same Killing vector ξ intersecting on a spacelike $(D-2)$-dimensional surface Σ, on which ξ vanishes. Σ is called the bifurcation surface. A most simple example is that of the Schwarzschild black hole in the Kruskal coordinate. The bifurcation surface is at its origin, and the Killing vector ξ acts as the Lorentz boost with strength κ near the origin.

Note that in the static coordinate system, any spacelike surface at constant t passes Σ. Thus the horizon cross section at any finite t *is* the bifurcation surface. Physically sensible spacetime with stationary black holes with nonzero constant κ is expected to have an extension in which the horizon becomes bifurcate. For more discussion, we refer the reader to [19] and the references therein.

Wald's formula (5). We will analyze them on a case-by-case basis in the next section.

As we saw, our algorithm depends on finding Σ_ξ etc., which has an arbitrariness of the form $\Sigma_\xi \to \Sigma_\xi + dX$. Before continuing, let us check that these ambiguities do not change the entropy. Firstly the shift just mentioned changes Q'_ξ by an exact form, which does not modify its integral over the horizon cross section. Secondly, the change of Ξ_ξ to $\Xi_\xi + dY$ induces $j \to j - \delta dY$, while it changes Σ_ξ to $\Sigma - \delta Y$. Thus Q'_ξ is left intact. Thirdly, the change in Q'_ξ caused by $\Theta \to \Theta + dZ$ can be seen to be proportional to $\delta_\xi \phi$ or ξ, which vanishes on Σ. Finally, the addition to the Lagrangian of a total derivative term $L \to L + dW$ results in the shift $\Theta \to \Theta + \delta W$ and $\Xi_\xi \to \Xi_\xi + \delta_\xi W$. Thus it does not change j_ξ or Σ_ξ, which means Q'_ξ is unchanged.

3 Examples

3.1 Three-dimensional gravitational Chern-Simons

Firstly let us consider three-dimensional gravity with Chern-Simons term

$$L_{CS} = \beta \operatorname{tr}(\Gamma R - \frac{1}{3}\Gamma^3), \tag{23}$$

where the wedge product should be understood. Since

$$(\delta_\xi - \pounds_\xi) L_{CS} = \beta \operatorname{tr} dU_\xi \cdot d\Gamma \tag{24}$$

where U_ξ is the matrix with component $(U_\xi)^\mu_\nu = \xi^\mu_{,\nu}$, we can choose $\Xi_\xi = -\beta \operatorname{tr} dU\Gamma$. Non-covariant contribution to the symplectic potential Θ is given by $-\beta \operatorname{tr} \Gamma\delta\Gamma$, hence $\Pi_\xi = -\beta \operatorname{tr} dU_\xi \delta\Gamma$. Thus for this choice $\Sigma_\xi = 0$ and

$$j_\xi = 2\beta \operatorname{tr} dU_\xi \Gamma + \text{ (terms linear in } \nabla\xi \text{ and } \xi). \tag{25}$$

Thus

$$Q'_\xi = 2\beta \operatorname{tr} U_\xi \Gamma + \text{ (terms linear in } \xi). \tag{26}$$

Evaluating $2\pi \int Q'|_{\xi\to 0, \nabla_a \xi_b \to \epsilon_{ab}}$ on the bifurcation surface, we arrive at the following formula

$$\Delta S_{CS} = 8\pi\beta \int_\Sigma \Gamma_N \tag{27}$$

where[3]

$$\Gamma_N = -\epsilon^\nu{}_\mu \Gamma^\mu{}_{\nu\rho} dx^\rho / 2 \tag{28}$$

[3]In our convention, the signature of the metric is mostly plus, and $\epsilon_{01\cdots}$ is positive. The binormal is defined so that ϵ_{01} is positive. Then $\epsilon^1{}_0$ and $\epsilon^0{}_1$ are negative.

is the projection of the affine connection to the normal bundle of Σ.

An important fact here is that the binormal ϵ_{ab} is covariantly constant on the bifurcation surface, because

$$\nabla_c \epsilon_{ab} = \kappa^{-1} \nabla_c \nabla_a \xi_b = \kappa^{-1} R_{abcd} \xi^d = 0. \tag{29}$$

It means that the holonomy of the metric on the bifurcation surface is reduced to lie in $SO(1,1)_N \times SO(D-2)_T$ where N, T stands for normal and tangential, respectively. The factor $\epsilon^\mu{}_\nu$ in (28) projects the connection to the normal component. Then $\int_\Sigma \Gamma_N$ is the one-dimensional Chern-Simons term for the $SO(1,1)$ connection of the normal bundle, and thus is gauge-invariant.

The formula (27) agrees with the formula obtained in [20] using the conical singularity method. For the rotating Bañados-Teitelboim-Zanelli black hole [9] with the metric

$$-\frac{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}{l^2\rho^2} d\tau^2 \\ + \frac{l^2\rho^2}{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)} d\rho^2 + \rho^2 (dy - \frac{\rho_+\rho_-}{l\rho^2} d\tau)^2 \tag{30}$$

which has the outer horizon at $\rho = \rho_+$, the contribution is

$$\Delta S = 8\pi\beta \int_\Sigma \frac{-\rho_+\rho_-}{lr} = -\frac{16\pi^2\beta\rho_-}{l}. \tag{31}$$

It agrees with the result given in the literature. The methods employed were diverse: it was done in [10, 11] via the consideration of the boundary Virasoro algebra, in [12] by dimensional reduction to two dimensions and application of the Wald's formula, in [14] using the description of three-dimensional gravity as the $SO(2,2)$ Chern-Simons theory, and in [15] by the direct integration of the first law.

3.2 Gravitational Chern-Simons in other dimensions

We can generalize the formula in the previous subsection to the following mixed gravitational Chern-Simons term

$$L_{CS} = (\operatorname{tr} \Gamma R^{2m-1} + \cdots) P(F) \tag{32}$$

where $P(F)$ is a closed form constructed out of fields other than the metric and $\cdots$ is determined from the descent relation $dL_{CS} = (\operatorname{tr} R^{2m}) P(F)$. The calculation which lead to (27) can be repeated almost verbatim to yield

$$\Delta S = 8\pi m \int_\Sigma \Gamma_N R_N^{2m-2} P(F), \tag{33}$$

where R_N is the curvature two-form for the normal bundle. All of the would-be contribution from the ellipsis $\cdots$ in (32) vanishes since $SO(1,1)_N$ is Abelian.

If there is a form ω such that $P(F) = d\omega$, the result (33) can be inferred from the original Wald's formula (5). Indeed, The term (32) then has the same effect with the term

$$L'_{CS} = \operatorname{tr} R^{2m} \wedge \omega, \tag{34}$$

where the affine connection does not appear explicitly. Thus we can apply the formula (5) to obtain the contribution to the entropy

$$\Delta S = 8\pi m \int_\Sigma R_N^{2m-1} \wedge \omega, \tag{35}$$

which is equivalent to the equation (33). Strictly speaking, ω needs to have a non-trivial gauge transformation between the coordinate patches on Σ if $P(F)$ has non-trivial flux through Σ. Thus one cannot naively partially integrate from (32) to (34) and one needs to be more careful. It would be interesting to rederive our formula using an auxiliary spacetime which has one extra dimension as is usually done for the correct definition of Chern-Simons terms in the presence of non-trivial fluxes.

3.3 Lagrangian of Green-Schwarz type

As another application of our analysis, let us determine the contribution to the entropy from the term in the six-dimensional Lagrangian of the form

$$L = \frac{1}{2g^2} H \wedge *H \tag{36}$$

where $H = dB + \lambda \operatorname{tr}(\Gamma R - \Gamma^3/3)$. In order to maintain the general covariance, the B field needs to have an extra term in the transformation under diffeomorphism given so that

$$\delta_\xi B = \pounds_\xi B - \lambda \operatorname{tr} dU_\xi \Gamma, \tag{37}$$

where U_ξ is the matrix $(U_\xi)^\mu_\nu = \xi^\mu_{,\nu}$ as before. Modified transformation law of the form-fields such as this often occurs in string theory in order to cancel the anomaly à la Green-Schwarz mechanism.

Now $\Xi_\xi = 0$ since L itself is covariant. The non-covariant portion of Θ is

$$\frac{1}{g^2}(\delta B - \lambda \operatorname{tr} \Gamma \delta\Gamma) \wedge *H \tag{38}$$

and thus $\Pi_\xi = -\lambda \operatorname{tr} dU\delta\Gamma/g^2$. Plugging into the formula for Q' (18), we obtain

$$Q'_\xi = \frac{2}{g^2}\lambda \operatorname{tr} U_\xi \Gamma + (\text{terms linear in } \xi). \tag{39}$$

Hence the contribution to the entropy is given by

$$\Delta S = 8\pi \frac{\lambda}{g^2} \int_\Sigma \Gamma_N \wedge *H. \tag{40}$$

One can also determine the same contribution, of course, by the use of the dualization and the formula (33) derived in the previous subsection. Indeed, we can dualize the term (36) to the form

$$L = \frac{g^2}{2} H_D \wedge *H_D - H_D \wedge \lambda \operatorname{tr}(\Gamma R - \frac{1}{3}\Gamma^3), \tag{41}$$

where $H_D = dB_D$ and $H_D = *H/g^2$ on-shell. Applying the formula (33) we obtain

$$\Delta S = 8\pi\lambda \int_\Sigma \Gamma_N \wedge H_D, \tag{42}$$

which agrees with (40) as it should be.

The formula obtained above can be checked against the entropy of the dyonic black hole treated in [13], where the entropy was determined by means of dimensional reduction. There, the Lagrangian of the theory contains a two-form with the kinetic term given by (36) and the theory is compactified on T^2. The black hole has a two-dimensional horizon cross section from the four-dimensional point of view, and one of the direction of T^2 is fibered non-trivially on it. Thus, the black hole has a four-dimensional horizon cross section of the topology $S^3 \times S^1$ if viewed as a six-dimensional spacetime. One can then evaluate the correction to the entropy using the expression (40) or (42) on the background, which was given in (3.4) of [13]. H_D has an integral flux through the S^3 part of the horizon, and then the correction reduces to the calculation of the one-dimensional Chern-Simons term on S^1. It reproduces the first term in (3.34) of [13]. The second term in (3.34) arose in [13] from the modification of the background geometry by the Chern-Simons interaction. In order to reproduce it from our formula (40) we will first need to determine the backreaction of the Chern-Simons term, which is left to a future work.

4 Summary and Outlook

In this short note, we studied how one can generalize the argument in [3, 4, 5] to obtain the entropy formula for the black holes in the presence of Chern-

Simons terms. We gave an explicit formula for several kinds of Chern-Simons terms. It correctly reproduced the contribution of the three-dimensional gravitational Chern-Simons term to the entropy, which had already been determined via other methods.

Concrete examples we saw above were both related to the BTZ black hole background, because the dyonic black hole in six dimensions reduces to the BTZ black hole if one treats S^3 part of the horizon as the internal space. Thus, it would be interesting to study the effect of Chern-Simons terms in other dimensionality, say, $L_{CS} = \mathrm{tr}\, \Gamma R^3 + \cdots$ for black holes in seven dimensions. Some string/M-theory compactification is known to have such interaction in seven dimensions, thus it would be interesting to consider brane construction for such black holes and check if the microscopic entropy matches with the macroscopic prediction here.

Another point is that Wald's formula and our generalized ones allow us to calculate the correction to the entropy from higher derivative terms only after we know the correction to the background from the same higher derivative terms. The entropy function formalism [21] devised by Sen is more convenient in that respect because it automatically incorporates both sources of corrections via the extremalization of the entropy function. Thus an interesting direction of research will be to generalize the said formalism to Lagrangians containing Chern-Simons terms.

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